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PIONEER PAPER IN HEAT AND MASS TRANSFER

A METHOD OF CORRELATING FORCED CONVECTION HEAT-TRANSFER DATA AND A COMPARISON WITH FLUID FRICTION*†

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Abstract—A general method for the correlation of forced convection heat-transfer data is proposed, which consists in plotting, against the Reynolds number, a dimensionless group representing the experimentally measured data from which film heat-transfer coefficients would be calculated, namely, $[(t_1 - t_2)/\Delta t_m](S/A)$, or its equivalent, h/cG, multiplied by the two-thirds power of the group, $(c\mu/k)$. Data are cited from the literature which show that the resulting plots of heat-transfer data for flow parallel to plane surfaces and for fully turbulent flow inside tubes, coincide (when the properties are taken at the "film" temperature) with the best data on fluid friction plotted in the customary manner, as the friction factor

$$\tfrac{1}{2}f = \frac{\Delta P g}{\rho u^2}\frac{S}{A} = \frac{R}{\rho u^2},$$

against the Reynolds number. For flow at right angles to tubes, however, the friction and heat-transfer factors differ, the friction factors being higher.

The equations successfully employed for representing heat-transfer data in streamline flow inside tubes have been modified for plotting with the same coordinates as used for turbulent flow; and a quantitative allowance is suggested for the effect of free convection at low velocities by including a function of the group, $(d^3\rho^2\beta\Delta t g/\mu^2)$. There is seen to be no relation between heat transfer and friction in the viscous region.

The method of correlation here proposed is shown to be particularly valuable in the transition region between streamline and turbulent flow in tubes, since heat-transfer factors may show "dips" analogous to those for friction. The controlling variables in this region are fully discussed in the light of the available data.

NOMENCLATURE

Throughout the paper, self-consistent units are used. In the following list, illustrative units are given in both the Metric and English systems, using meter, kilogram, hour, °C, and kilogram-calorie in the former, and foot, pound, hour, °C, and pound-Centigrade heat unit in the latter.

- A, surface area [m²], [ft²];
- G, [kg/h m²], [lb/h ft²];
- G_m, maximum mass velocity (through minimum section) [kg/h cm²], [lb/h ft²];
- Gr, Grashof number;
- L, heated length in direction of flow [m], [ft];
- ∂L, differential length [m], [ft];
- N, number of rows of tubes in the direction of flow;
- ΔP, pressure drop [kg/m²], [lb/ft²];
- R, frictional resistance, force units per unit surface area [(kg/m²)(m/h h)], [(lb/ft²)(ft/h h)];
- Re, Reynolds number;
- S, cross-sectional area [m²], [ft²];
- W, weight flow rate [kg/h], [lb/h];
- a, constant;

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b, constant;
c, c_p, specific heat (at constant pressure for gases) [kg cal/kg degC], [P.c.u./lb degC];
c_v, heat capacity at constant volume [kg cal/kg degC], [P.c.u./lb degC];
d, inside or equivalent hydraulic diameter [m], [ft];
d_p, outside diameter of pipe [m], [ft];
d_s, clearance between tubes in a row normal to direction of flow [m], [ft];
f, friction factor;
g, acceleration due to gravity [m/h h], [ft/h h];
h, film coefficient of heat transfer [kg cal/h m^2 degC], [P.c.u./h ft^2 degC];
j, heat-transfer factor;
k, thermal conductivity [kg cal/h m^2] (degC/m), [P.c.u./h ft^2] (degC/ft);
k_d, diffusion coefficient [m^2/h], [ft^2/h];
m, constant;
n, constant;
∂p, differential pressure force units [(kg/m^2)(m/h h)], [(lb/ft^2)(ft/h h)];
r, factor in Prandtl equation;
t, temperature [°C], [°C];
t_a, average fluid temperature [°C], [°C];
t_w, average wall temperature [°C], [°C];
t_f, film temperature $= t_a + \frac{1}{2}(t_w - t_a)$ [°C], [°C];
t_{vf}, film temperature for viscous flow $= t_a + \frac{1}{4}(t_w - t_a)$ [°C], [°C];
Δt_m, mean temperature difference across film [°C], [°C];
u, linear velocity [m/h], [ft/h];
u_m, maximum velocity (through minimum section) [m/h], [ft/h];
β, coefficient of thermal expansion [1/degC], [1/degC];
ρ, density [kg/m^3], [lb/ft^3];
μ, μ_f, viscosity at film temperature [kg/h m], [lb/h ft];
μ_a, viscosity at average temperature [kg/h m], [lb/h ft].

Dimensionless groups
dG/μ, Reynolds number* for flow in conduits;
d_pG/μ, Reynolds number for flow across pipes;
LG/μ, Reynolds number for flow parallel to plane surfaces;
hd/k, Nusselt number;*
$c\mu/k$, Prandtl number;*
dcG/k, Peclet number;*
Wc/kL, Graetz number;†
$d^3\rho^2\beta\Delta tg/\mu^2$, Grashof number;*
$\mu/\rho k_d$, Schmidt number;‡
h/cG, Stanton number.‡

ALTHOUGH great strides have been made in the correlation of forced convection heat-transfer data in recent years, the state of knowledge has not been entirely satisfactory because of the large number of different equations and plots necessary to treat the various types of apparatus, flow conditions encountered, and fluids used, and also because the often-mentioned possible relationship with fluid friction has not been conclusively demonstrated or its limits clearly defined. It is the purpose of this paper to simplify the field of forced convection by introducing a general method of correlating heat-transfer data which can be used for the entire range of turbulent and viscous flow in various types of apparatus, and which results in a strikingly direct comparison with friction data.

The method of correlation proposed is to plot $(h/cG)(c\mu/k)^{2/3}$ versus dG/μ, where h is the film coefficient of heat transfer between fluid and solid, i.e. the quantity of heat transferred per unit time, unit surface area, and unit temperature difference, c is the specific heat of the fluid (if a gas, at constant pressure), G is the weight velocity, i.e. the weight of fluid flowing per unit time and unit cross-sectional area, μ is the

* Names adopted by "Ausschuss für Wärmeforschung im Verein deutscher Ingenieure" [23] and used by McAdams [36].

† Name suggested by McAdams [36].

‡ Name proposed at Round Table Conference of Chicago meeting American Institute of Chemical Engineers, 15 June 1933.

viscosity of the fluid, and d is the characteristic linear dimension such as diameter. The groups (h/cG), $(c\mu/k)$ and (dG/μ) are all dimensionless so that any self-consistent set of units can be used. These groups are also known as Stanton, Prandtl and Reynolds (St, Pr and Re) numbers. It can be readily seen from the definitions of h and G that the group (h/cG) is equal to the expression $[(t_1 - t_2)/\Delta t_m](S/A)$, where $t_1 - t_2$ is the temperature change, Δt_m is the mean temperature difference between the fluid and the surface, A is the heat transfer area and S is the cross-sectional area for flow.

As explained in the appendix to this paper, this procedure of correlation has its basis in the Reynolds analogy, but includes a function of $c\mu/k$ to correct for differences between the temperature and velocity distributions. According to this modified analogy, and ordinate given above is, under certain conditions, equal to one-half the friction factor, f, which can be defined in terms of either the overall pressure drop or the frictional resistance as follows:*

$$\tfrac{1}{2}f = \frac{\Delta P g}{\rho u^2}\frac{S}{A} = \frac{R}{\rho u^2}, \qquad (1)$$

where ΔP is the pressure drop (in weight units) per unit cross-sectional area, g is the acceleration of gravity, ρ is the density of the fluid, u is the average linear velocity of the fluid, and R is the frictional resistance (in force units) per unit surface area. A corresponding heat-transfer factor, j, can be defined in terms of either the overall temperature change or the heat-transfer coefficient:

$$j = \frac{t_1 - t_2}{\Delta t_m}\frac{S}{A}\left(\frac{c\mu}{k}\right)^{2/3} = \frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3}. \qquad (2)$$

Under conditions where the modified Reynolds analogy holds, j is equal to $\frac{1}{2}f$, but under other conditions there is no equality between these two factors and different symbols are therefore chosen to represent them.

Besides presenting a direct comparison of heat transfer and friction, this method of plotting data has another advantage which is best understood by a comparison with the most popular previously used type, where $(hd/k)/(c\mu/k)^{1/3}$ was plotted versus dG/μ. That the two methods are very similar is evident from the relationship between the old and new ordinates:

$$\left[\frac{hd}{k}\Big/\left(\frac{c\mu}{k}\right)^{1/3}\right] = \left[\frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3}\right]\left[\frac{dG}{\mu}\right] = \left[\frac{t_1 - t_2}{\Delta t_m}\frac{S}{A}\left(\frac{c\mu}{k}\right)^{2/3}\right]\left[\frac{dG}{\mu}\right]. \qquad (3)$$

Plotting the old ordinate versus Reynolds number was the same, however, as plotting

$$\left[\frac{t_1 - t_2}{\Delta t_m}\frac{S}{A}\left(\frac{c\mu}{k}\right)^{2/3}\right]\left[\frac{dG}{\mu}\right]$$

against dG/μ, which thus involved plotting a function against itself, when it is considered that the experimental data are given by the expression

$$\frac{t_1 - t_2}{\Delta t_m}\frac{S}{A}\left(\frac{c\mu}{k}\right)^{2/3}$$

and that values of this function do not vary widely as compared with variations in dG/μ. It is of interest that this ordinate is essentially the one introduced by Reynolds, as shown in the Appendix.

A helpful feature of the proposed method of presenting data is that the value of the ordinate is a direct function of the temperature change in a heat exchanger, and the effect of varying the velocity of flow in the exchanger on the exit temperature is indicated at once. Furthermore, for given temperature conditions, the design of heating surface is seen to be practically set by the ratio of surface area to cross-sectional area; for flow inside tubes, for example, this ratio is proportional to the length divided by the diameter.

The function of $c\mu/k$ here employed was obtained from previous correlations of Morris and Whitman [42], Hinton [21], Cox [9], and Sherwood and Petrie [56]. While the exponents of the $c\mu/k$ group proposed by these workers ran from 0·3 to 0·4 on the old ordinates, which would be from 0·7 to 0·6 on the new, the 0·66 or $\frac{2}{3}$ power was chosen because it is more or less of an average value. Since the theoretical Prandtl equation, discussed in the Appendix, utilizes a different function of $c\mu/k$, it is of interest to show how the functions differ. A comparison is given by Fig. 1, which indicates that the function used

* There is no significance in the use of $\frac{1}{2}f$ rather than f to represent $R/\rho u^2$, other than the fact that this symbol has customarily been so defined in this country [69].

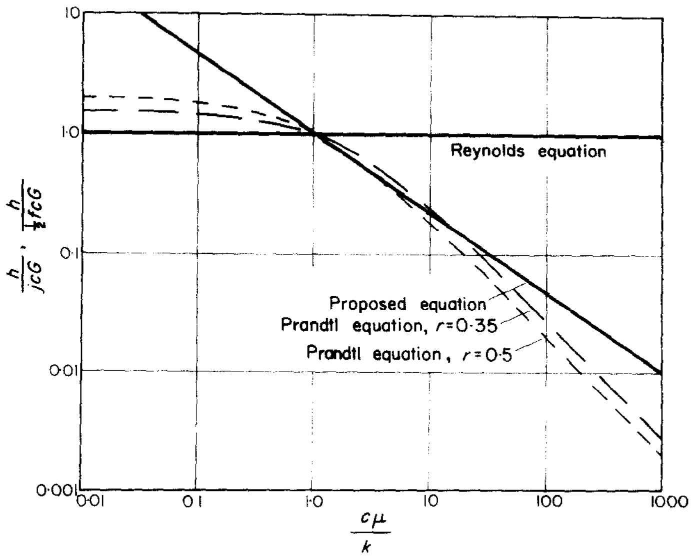


FIG. 1. Variation of heat-transfer coefficient with $(c\mu/k)$ as predicted by various equations.

Reynolds equation: $h = \frac{1}{2} fcG$.

Prandtl equation: $h = \frac{\frac{1}{2} fcG}{(1 - r) + r(c\mu/k)}$

Proposed equation: $h = \frac{jcG}{(c\mu/k)^{2/3}}$.

herein and that of the Prandtl equation are nearly the same from $c\mu/k = 1$ to $c\mu/k = 10$, but that for higher values, the Prandtl predicts lower results. It is thus apparent that data on water, where $c\mu/k$ runs from 2 to 10, could hardly be used to determine a choice between the Prandtl and the proposed equations, but there should be little doubt for values of $c\mu/k$ above 100, i.e. for viscous oils.

Applications of the proposed method to flow inside of tubes, flow across single tubes and tube banks, and flow parallel to plane surfaces will be discussed separately below.

1. HEAT TRANSFER AND FLUID FRICTION INSIDE TUBES

Since the mechanism of heat transfer is dependent on the flow conditions, it is a helpful preliminary step in the study of heat transfer inside tubes to outline the effects of flow as indicated by data on fluid friction.

Fluid friction. When data on pressure drop under conditions of *isothermal flow* are plotted as friction factor, as defined by Equation 1, versus Reynolds number, dG/μ, there are three distinct regions indicated by the data: First, at Reynolds numbers less than 2300, the data fall on or near a straight line which represents Poiseuille's law as given by the equation:

$$\tfrac{1}{2}f = \frac{\Delta P g}{\rho u^2}\frac{S}{A} = 8\left(\frac{dG}{\mu}\right)^{-1}. \qquad (4)$$

Secondly, from a Reynolds number of 2300 to one of about 3000, the value of friction factor rises about 50 per cent, so that at 2300 there appears a considerable "dip". Thirdly, at Reynolds numbers greater than about 3000, the data fall in a band which can be represented by a smooth curve yielding decreasing values of friction factor with increasing Reynolds numbers. Visual observations have shown that below a Reynolds number of 2300, the flow is streamline or viscous, and that above, it is turbulent. Drew, Koo and McAdams [14] have made an extensive correlation of data on friction for turbulent flow, and have found that the band of data for smooth pipes at Reynolds numbers greater than 3000 can be represented with less than ± 10 per cent deviation by the equation:

$$\tfrac{1}{2} f = 0{\cdot}0007 + 0{\cdot}0625\,(dG/\mu)^{-0\cdot32} \quad (5)$$

When the pipe wall is being heated or cooled so that there is a temperature gradient through the fluid in the pipe, it has been shown by data of Keevil and McAdams [26] that in the viscous region, equation (4) can be used if the viscosity is taken at a film temperature, t_{vf}, defined as follows:

$$t_{vf} = t_a + \tfrac{1}{4}\,(t_w - t_a), \quad (6)$$

where t_a is the average fluid temperature in the length of pipe under consideration, and t_w is the average wall temperature; and that in the turbulent region, at least for Reynolds numbers greater than 7000, equation (5) holds if the viscosity is taken at a film temperature, t_f, defined as follows:

$$t_f = t_a + \tfrac{1}{2}\,(t_w - t_a). \quad (7)$$

Keevil and McAdams first plotted their non-isothermal data as friction factor versus Reynolds number in which the viscosity was taken at t_a, and obtained a series of curves for various temperature differences which were similar in shape to that for isothermal flow. It now appears that a particularly significant feature of this plot is, that the transition from viscous to turbulent flow, as indicated by the lower part of the dip, occurs at a value of Reynolds number of 2300 based on the viscosity, μ_a, at the average fluid temperature, t_a, even though the friction is dependent on the viscosity at the film temperature. It is thus apparent that when the data are plotted so that they can be extrapolated to any temperature conditions by using film-temperature Reynolds numbers, the line for the viscous region will extend to $Re = 2300\,\mu_a/\mu_{vf}$, and that the curve in the turbulent region will begin at $Re = 2300\,\mu_a/\mu_f$. It will therefore be appreciated that for heating liquids, the viscous line will extend considerably farther than a film Reynolds number of 2300, and therefore the dip of the friction factor will be accentuated, whereas for cooling, the transition from viscous to turbulent will occur before $Re = 2300$, and the friction factor will not go through so great a dip as that in isothermal flow. Since the dip region cannot be conveniently expressed in a formula, problems in this range are best solved from plots. Equations (4) and (5) and dips under conditions of heating and cooling for various ratios of μ_a/μ_f are shown by the dashed lines on the résumé chart given by Fig. 16.

Heat transfer. In utilizing the proposed method of correlation for heat transfer inside tubes, it was expected that there would be found distinctive regions in the same ranges of Reynolds numbers as for fluid friction. Therefore an attempt was made to cover the widest possible range of conditions reported in the literature, especially for the dip region.

In the *turbulent region*, at high enough Reynolds numbers to ensure the data being out of the dip, several of the most reliable sets of published results were correlated as shown later, after recalculation of the viscosity to a film temperature, t_f, and the points both for heating and for cooling were found to fall very close to the friction factor line represented by equation (5), thus showing for this region complete agreement with the modified Reynolds analogy. There results the following equation for heat transfer for turbulent flow in pipes:

$$i = (h/cG)\,(c\mu_f/k)^{2/3} = 0{\cdot}0007 + 0{\cdot}065\,(dG/\mu_f)^{-0\cdot32}. \quad (8)$$

This equation can be approximated by the following formula:

$$j = (h/cG)\,(c\mu_f/k)^{2/3} = 0{\cdot}023\,(dG/\mu_f)^{-0\cdot2}. \quad (9)$$

As shown by equation (3), this formula can be expressed in the old manner by multiplying both sides by Reynolds number, which gives:

$$(j)\,(dG/\mu_f) = (hd/k)/(c\mu_f/k)^{1/3} = 0{\cdot}023\,(dG/\mu_f)^{0\cdot8}. \quad (9a)$$

It will be noted that expressed as above, the present results are quite comparable with the equation for heating during turbulent flow in pipes recommended by McAdams [38]:

$$(hd/k)/(c\mu_a/k)^{0\cdot4} = 0{\cdot}0225\,(dG/\mu_a)^{0\cdot8}, \quad (10)$$

where the viscosity is taken at the average fluid temperature, t_a.

In the *viscous region* it was found that correlations of previous investigators could be utilized by rearranging the equations obtained from the data. McAdams [39] has recorrelated most of the reliable data from the literature according to the theoretical equation of Graetz [20], modifications of which have recently been discussed in the light of experimental data by Kirkbride and McCabe [27] and Drew, Hogan and McAdams [13]. The Graetz equation, which assumes a parabolic velocity distribution and absence of free convection currents, can be approximately represented, for values of Wc/kL greater than 10, by the equation:

$$\frac{h_a d}{k} = 1{\cdot}65 \left(\frac{Wc}{kL}\right)^{1/3}, \qquad (11)$$

where h_a is the heat-transfer coefficient based on an arithmetic mean temperature difference and W is the weight flow per unit time. This equation can be rearranged to solve for the newly defined heat-transfer factor as follows:

$$j = \frac{h_a}{cG}\left(\frac{c\mu}{k}\right)^{2/3} = 1{\cdot}5 \left(\frac{dG}{\mu}\right)^{-2/3} \left(\frac{L}{d}\right)^{-1/3}. \qquad (11a)$$

In this form, the viscosity is included to the same power in both sides of the equation and will cancel, hence it is immaterial whether it is taken at t_{vf} as for friction in the viscous region or at t_f or even at t_a. McAdams' correlations show that data on heating are from 50 to 120 per cent higher than the theoretical equation, which he ascribed to free convection, and that data on cooling are about equally above and below the theoretical equation.

It was pointed out by Colburn and Hougen [8] that heat-transfer data in the range of viscous flow, and sometimes in the lower range of Reynolds numbers for turbulent flow, would be influenced markedly by the Grashof number, $(d^3\rho^2\beta\Delta t g/\mu^2)$, where β is the coefficient of thermal expansion of the fluid (equal to the reciprocal of the absolute temperature for gases). However, the Grashof numbers are about the same for cooling as for heating so that while the effect of free convection might explain some of the deviation of the data from the theoretical curve it cannot explain the wide divergence between the heating and cooling values. Furthermore, data of Drew [12] on the heating of glycerine in a tube of small diameter indicate a greater increase in heat-transfer coefficient with increase in temperature difference than can be explained entirely by increased free convection. This divergence, therefore, undoubtedly results from the difference in the velocity distributions, as pictured by Keevil and McAdams [26], owing to greater viscosity of the liquid near the pipe surface than in the center of the pipe during cooling, and decreased viscosity near the surface during heating. This effect must be a function of the viscosity change across the cross-section, which can be relatively expressed by the magnitude of the ratio, μ_a/μ_f. By including this factor to the $\frac{1}{3}$ power in equation (11), it has been found possible to bring the data for heating and cooling into approximate agreement. To evaluate the free convection effect, data at high Grashof numbers were necessary; these were noted in the compressed air runs of Nusselt [44] at low Reynolds numbers and in the results on water in a large diameter pipe of Colburn and Hougen [8]. Plots of these data, shown later by Figs. 3 and 7, were compared with equation (11), after including a factor of $(\mu_a/\mu_f)^{1/3}$, ranging from 1·15 to 1·27 for the Colburn and Hougen data. Lines drawn through the points of similar Grashof numbers deviated from the theoretical line representing equation (11), and the ratios of the observed to theoretical values of j were then plotted versus Grashof number as shown on Fig. 2. A point was also included for a deviation of 27 per cent at a Grashof number of 4000 to represent an average deviation of the oil data given on the plots of McAdams from the theoretical equation after a μ_a/μ_f correction had been applied. The equation representing the line drawn through the data is as follows:

$$j(\text{act})/j(\text{theoret}) = (1 + 0{\cdot}015\, Gr^{1/3}). \qquad (12)$$

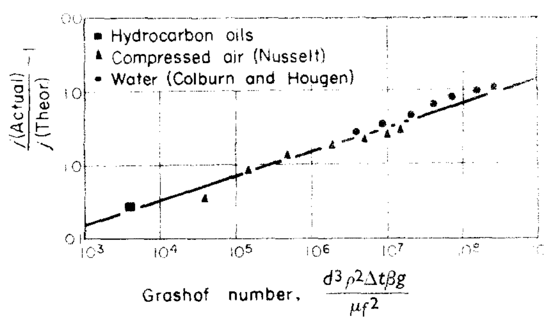


FIG. 2. Effect of free convection group on heat transfer in viscous flow.

Equation (11) can then be amplified to include both the effects of viscosity changes and free convection as follows:

$$\frac{h_a d}{k} = 1{\cdot}65 \left(\frac{Wc\phi}{kL}\right)^{1/3}, \quad (13)$$

$$j = \frac{h_a}{cG}\left(\frac{c\mu}{k}\right)^{2/3} = 1{\cdot}5 \left(\frac{dG}{\mu}\right)^{-2/3} \left(\frac{L}{d}\Big/ \phi\right)^{-1/3}, \quad (14)$$

where

$$\phi = (\mu_a/\mu_f)(1 + 0{\cdot}015\, Gr^{1/3})^3, \quad (15)$$

$$Gr = (d^3 \rho^2 \Delta t \beta g/\mu_f^2).$$

The factor, ϕ, is chosen to affect the heat-transfer coefficient only as the cube root in order that it can be conveniently represented graphically together with the term L/d. Typical values of Gr and ϕ are given in Table 1, and to indicate the magnitude of the effect on the heat-transfer coefficient, values of $\phi^{1/3}$ are also included. It should be noted that equation (15) represents only an approximation based upon the few data at present available and that further data are necessary to make possible a more accurate expression. On the following plots of the data of Nusselt and of Colburn and Hougen, where Gr varies over wide limits, lines predicted by equation (14) are given for various values of Gr.

For values of Wc/kL less than 10, Drew, Hogan, and McAdams [13] showed that for cases of *constant pipe-wall temperatures*, the exit fluid is practically at the pipe wall temperature, so that if the heat-transfer coefficient is based on an arithmetic mean temperature difference, the data tend to approach as a maximum an asymptote expressed by the equation:

$$\frac{h_a d}{k} = \frac{2}{\pi}\frac{Wc}{kL} \quad (16)$$

or upon rearranging

$$j = \frac{h_a}{cG}\left(\frac{c\mu}{k}\right)^{2/3} = 0{\cdot}5 \left(\frac{L}{d}\right)^{-1} \left(\frac{c\mu}{k}\right)^{2/3}, \quad (17)$$

Such an equation is represented by horizontal lines on a plot of j versus dG/μ.

The most interesting feature of the new correlations is the appearance of the data in the *dip region*. It was found that the location of the data depended on the ratio of μ_a/μ_f, just as for friction in the dip region, and also on the ratio of length to diameter. Since a simple equation could not be developed for these relationships, it was felt

Table 1. Typical values of ϕ for $t_f = 62°C$, $\Delta t = 25$ degC

Fluid	Diameter (cm)	Diameter (in)	Gr	$(1 + 0{\cdot}015\, Gr^{1/3})$	Heating μ_a/μ_f	Heating ϕ	Heating $\phi^{1/3}$	Cooling μ_a/μ_f	Cooling ϕ	Cooling $\phi^{1/3}$
Air, 1 atm.	1·25	0·5	4000	1·24	—	1·9	1·24	—	1·9	1·24
1 atm.	2·5	1·0	31 000	1·49	—	3·3	1·49	—	3·3	1·49
1 atm.	5·0	2·0	248 000	1·93	—	7·2	1·93	—	7·2	1·93
10 atm.	2·5	1·0	5 000 000	3·56	—	45	3·56	—	45	3·56
Water	0·63	0·25	88 000	1·67	1·25	5·7	1·80	0·8	3·6	1·50
	1·25	0·5	700 000	2·33	1·25	15·6	2·50	0·8	10·0	2·15
	2·5	1·0	5 600 000	3·67	1·25	61	3·94	0·8	39	3·40
	5·0	2·0	44 800 000	6·32	1·25	310	6·75	0·8	200	5·85
Gas oil	1·25	0·5	66 000	1·61	1·2	4·8	1·69	0·83	3·3	1·49
	2·5	1·0	530 000	2·21	1·2	13	2·35	0·83	9·0	2·08
	5·0	2·0	4 250 000	3·41	1·2	81	4·35	0·83	56	3·83
Light H. T. oil	1·25	0·5	3700	1·23	1·52	2·8	1·40	0·66	1·2	1·06
	2·5	1·0	29 000	1·46	1·52	4·7	1·68	0·66	2·0	1·26
	5·0	2·0	235 000	1·93	1·52	10·9	2·20	0·66	4·7	1·68

(Gas oil (42), $\rho_{20°} = 52{\cdot}5$ lb/ft^3, $\beta = 0{\cdot}0007$/degC, $\mu_{20°} = 9{\cdot}7$, $\mu_{55°} = 4{\cdot}8$, $\mu_{100°} = 3{\cdot}7$ lb/h ft; Light heat transfer oil (55), $\rho_{20°} = 57{\cdot}5$ lb/ft^3, $\beta = 0{\cdot}0007$/degC, $\mu_{20°} = 145$, $\mu_{55°} = 24$, $\mu_{100°} = 8$ lb/h ft; water, $\beta_{62°} = 0{\cdot}0003$/degC.)

that the most expedient method of formulating general information for this region was by the preparation of a résumé chart on which the far turbulent region could be represented by a curve of equation (8), the viscous region by lines representing equations (14) and (17), and the dip region by curves estimated from plots of all the available data in this region. Such a chart is given in Fig. 16. To show how the plotted experimental data agree with this résumé chart, lines are included on the plots of data which represent what would be predicted by the chart for average experimental values of the variables involved.

Correlations of experimental data from the literature. The data chosen from the literature for heat transfer inside tubes are summarized in Table 2.

A comparison of equation (8) for heat transfer in the *turbulent* region with data is given by Figs. 3 to 6, 8 to 15, inclusive. It can be seen from the plots that the line representing equation (8) shows satisfactory agreement with most of the data beyond the dip region; about as many points lie above the line as below. Figures 3, 6, 8, 9 and 12 also show friction data taken the same apparatus as the heat transfer, and these points are in excellent agreement with the line which also represents equation (5). It should be stated that the high pressure runs of Poensgen [46] on cooling steam, shown by Fig. 4, were not corrected for radiation, which may account for their being so much higher than the curve. The points shown on Fig. 6 for Eagle and Ferguson's [15] experiments were taken from curves of their smoothed data, as they unfortunately did not include their original data. Only a portion of Sherwood and Petrie's [56] runs on water was plotted on Fig. 6—every fifth run was taken from their table of original data. Since the water temperature changed considerably in passing through the pipe in their experiments the temperature used for the correlation was that allowing for a changing heat-transfer coefficient by a method suggested previously [7]. Burbach's [3] data on Fig. 13 are about 100 per cent higher than the curve, but these data have previously been shown to be unusually high by Lawrence and Sherwood [32]; they were included herein only to exemplify the dip region. Inasmuch as the oil data for both heating and cooling plotted

Table 2. Summary of data plotted for flow inside tubes

Investigator	Ref.	Fluid	Diam. cm.	L/d	Heating $c\mu/k$	Heating μ_a/μ_f	Cooling $c\mu/k$	Cooling μ_a/μ_f
Nusselt	44	Air	2·21	27	0·76			
Nusselt	44	Carbon dioxide	2·21	27	0·83			
Josse	24	Air	2·31	58	0·76			
Poensgen	46	Superheated steam	3·94, 9·59	99, 41			1·17	
Colburn and Hougen	8	Water	7·8	24	2–8			
Eagle and Ferguson	15	Water	1·3–3·8	—	2–6			
Sherwood and Petrie	56	Water	1·26	97	1·8–3			
Burbach	3	Water	0·5	25–400			1·5–10	
Morris and Whitman	42	Water	1·57	196	2·1–2·4			
Morris and Whitman	42	Gas oil	1·57	196	25–29	1·3–1·6	22–41	0·72–0·84
Morris and Whitman	42	Straw oil	1·57	196	40–48	1·4–2·8	32–230	0·45–0·67
Morris and Whitman	42	Light motor oil	1·57	196	85–160	2·4–3·1	200–740	0·3–0·45
Keevil and McAdams	26	Velocite B oil	1·26	110	67–100	1·6–4·9		
Keevil and McAdams	26	Rabbeth I oil	1·26	110	52–56	1·9–2·4	85–190	0·59–0·68
Kraussold	28	Machine oil	2·67	117			300–500	0·43–0·72
Kraussold	28	Transformer oil	2·67	117			100–130	0·66–0·83
Sherwood, Kiley and Mangsen	55	Light heat transfer oil	1·5	61–234	68–90	1·4–2·2		

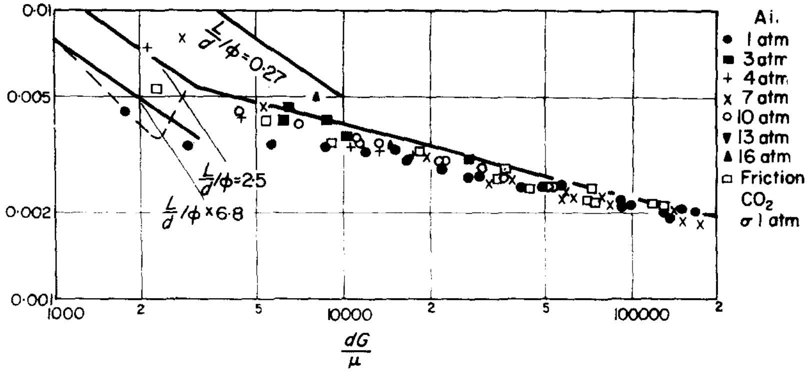


FIG. 3. Nusselt—heating air and carbon dioxide. $L/d = 27$.

Air—1 atm, $Gr = 40\,000$, $\phi = 4$, $(L/d)/\phi = 6{\cdot}8$.
Air—4 atm, $Gr = 500\,000$, $\phi = 11$, $(L/d)/\phi = 2{\cdot}5$.
Air—16 atm, $Gr = 15\,000\,000$, $\phi = 100$, $(L/d)/\phi = 0{\cdot}27$.

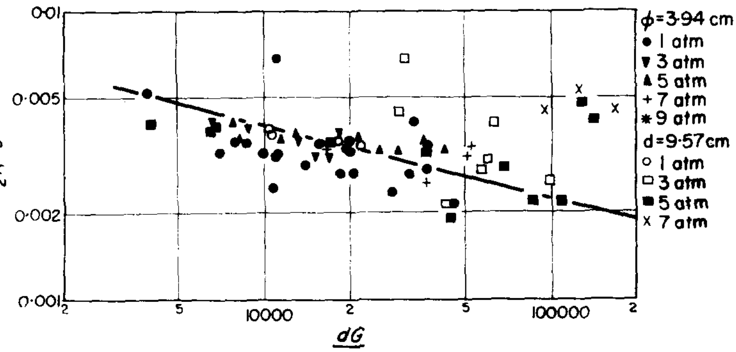


FIG. 4. Poensgen—cooling superheated steam.

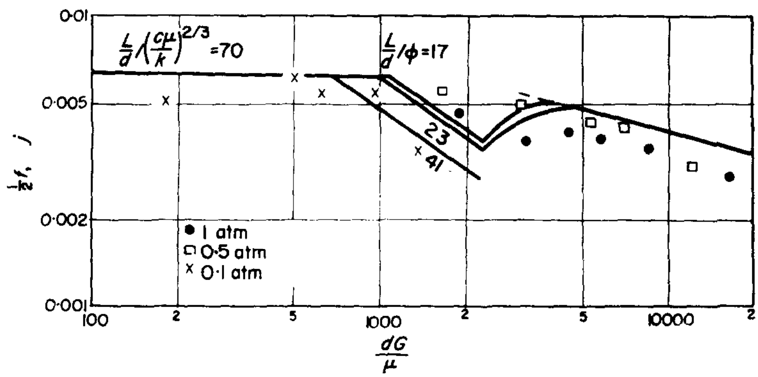


FIG. 5. Josse—heating air. $L/d = 58$.

1·0 atm, $Gr = 40\,000$, $\phi = 3{\cdot}5$, $(L/d)/\phi = 17$.
0·5 atm, $Gr = 10\,000$, $\phi = 2{\cdot}5$, $(L/d)/\phi = 23$.
0·1 atm, $Gr = 400$, $\phi = 1{\cdot}4$, $(L/d)/\phi = 41$.

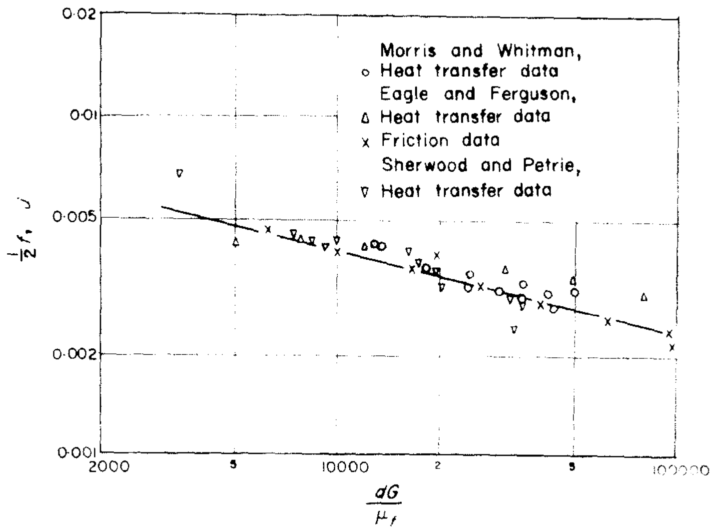


FIG. 6. Water in tubes. Heat transfer and friction.

on Figs. 8–12, 14 and 15 indicate fair agreement with equation (8) at Reynolds numbers high enough to be out of the dip, both the use of film temperatures and the particular function of $c\mu/k$ are considered to be substantiated by these results. It can now be explained why Morris and Whitman [42] found their data for heating and cooling oils could not be brought into agreement by use of the film temperature, t_f, since most of their heating data were in the dip region as shown by Fig. 11 and therefore were considerably below the usual turbulent line.

The *dip region* is well exemplified for liquids of various viscosities by Figs. 8–11 for heating and Figs. 12–15 for cooling. Although the data of each investigator cover a range of values of temperature difference and μ_a/μ_f, which would mean varying values of ϕ, lines are shown, for average values of ϕ, predicted from the résumé chart, Fig. 16. The dip region is barely suggested on Figs. 3 and 5 for heating air. While there are not sufficient data available at present to locate their position definitely, it is felt that interpolation among dip lines shown on the résumé chart will be satisfactory for approximation purposes.

The application of equation (14) for *viscous flow* and *free convection* conditions is shown by predicted lines on Figs. 3, 5, 7–10, 12–15. Figure 3 indicates that even in turbulent flow, conditions which cause high Grashof numbers may result in higher heat-transfer coefficients than predicted by equation (8).

Résumé charts for fluids in tubes

A résumé chart is given by Fig. 16 for both *heating* and *cooling*, which should permit the solution of heat-transfer and friction problems in the viscous, dip, and turbulent regions. The recommended heat-transfer curve for the turbulent region is a representation of equation (8) and is identical with the friction line representing equation (5). The heat-transfer lines in the viscous region are obtained from equation (14); the asymptote lines for cases of constant surface temperature come from equation (17). For the dip region, curves were drawn as suggested by Figs. 7–14, the curves leaving the viscous lines at $dG/\mu_f = 2300\,\mu_a/\mu_f$. The friction line for the viscous region is obtained from equation (4).

In using this chart the following features should be remembered:

Mean fluid temperature. The mean fluid temperature can be taken as the average of inlet and outlet temperatures when the temperature rise or fall is small. For other cases, the correct temperature can be obtained from a chart previously published [7].

Film temperature. In *heat-transfer* calculations, the average film temperature, t_f, defined by equation (7), should be used in determining the

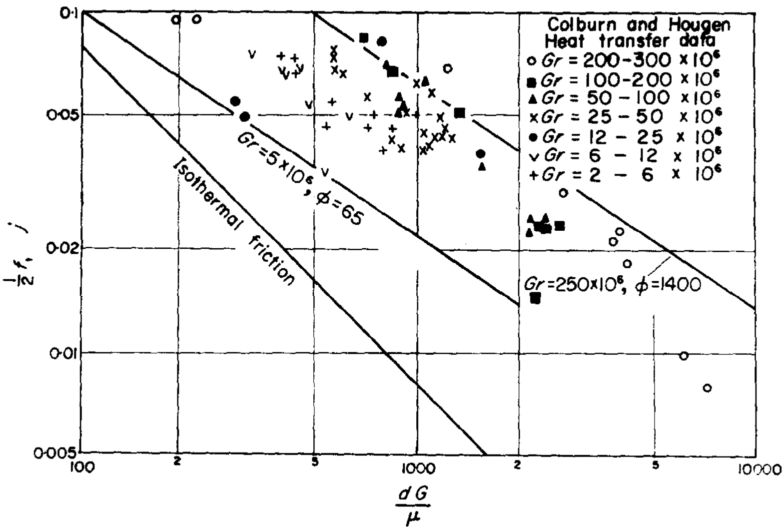
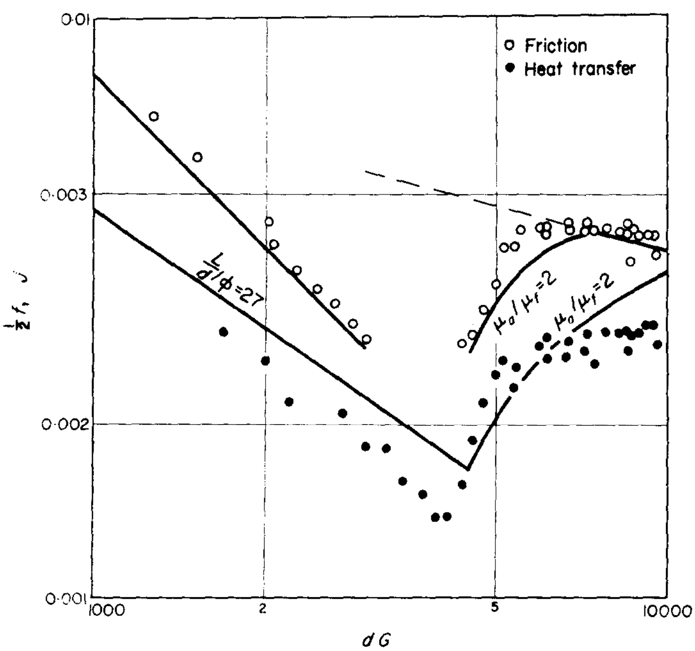


FIG. 7. Water in tubes, low velocities. Data of Colburn and Hougen for upward flow in a 3-in vertical pipe, $L/d = 24$.



FIGS. 8, 9, 10, 11. Oils in tubes—heating.

FIG. 8. McAdams and Keevil—heat-transfer and friction data.

Rabbath I oil $(c\mu/k) = 52\text{–}56$, $\phi \doteq 4$.
$L/d = 110$ $(\mu_a/\mu_f) = 1{\cdot}9\text{–}2{\cdot}4$.

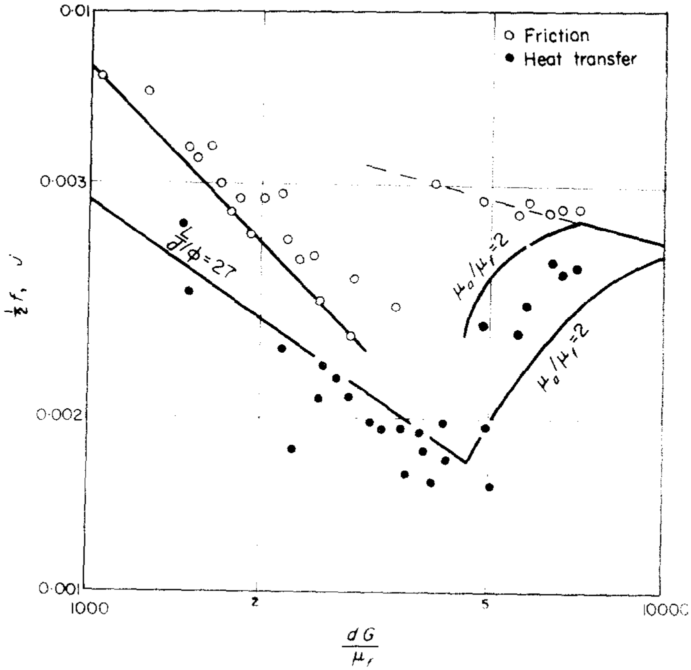


FIG. 9. McAdams and Keevil—heat-transfer and friction data

Velocite B oil $(c\mu/k) = 67\text{–}100$, $\phi \doteq 4$.
$L/d = 110$ $(\mu_a/\mu_f) = 1{\cdot}6\text{–}4{\cdot}9$.

viscosity. In *friction* calculations the average film temperature, t_f, should be used when the Reynolds number on a mean fluid temperature basis is greater than 2300; for lower Reynolds numbers, the film temperature, t_{vf}, defined by equation (6), should be used.

Mean temperature difference. When the Reynolds number on a mean fluid temperature basis is greater than 2300, the logarithmic mean temperature difference should be used; when less, the arithmetic mean.

Procedure in using chart. First obtain the mean fluid temperature, t_a, the average film temperature, t_f, and for these temperatures the ratio, μ_a/μ_f. Calculate the Reynolds number on both the μ_a and μ_f bases. If the Reynolds number on the μ_a basis is *greater* than 2300, the flow is turbulent; the log mean temperature should be used, and the heat transfer and friction factors will be located either on an interpolated μ_a/μ_f line, or on the main turbulent curve at an abscissa of Reynolds number based on μ_f, except for cases of large Grashof numbers. If the mean-temperature Reynolds number is less than 2300, the flow is streamline; the arithmetic mean temperature difference should be used. The value of $(L/d)/\phi$ should be computed, which will probably require an estimation of L. It should be noted that L is the heated or cooled length of tube before mixing occurs, not the composite length of several tubes in series. At the respective film Reynolds numbers, the heat-transfer and friction factors can then be read from the résumé chart. For *constant surface temperature*, the lines, $(L/d)/(c\mu/k)^{2/3}$ represent maximum values which the heat-transfer factor, j, cannot exceed.

One of the main values of this chart is that it supplies a procedure for predicting heat transfer in the dip region. Where a number of viscous flow heating problems are to be solved, equation (13) is simpler to use than Fig. 16, but where there is doubt as to the range of Reynolds number covered, Fig. 16 will be useful. Since, as previously mentioned, the heat-transfer ordinate of this chart is equal to

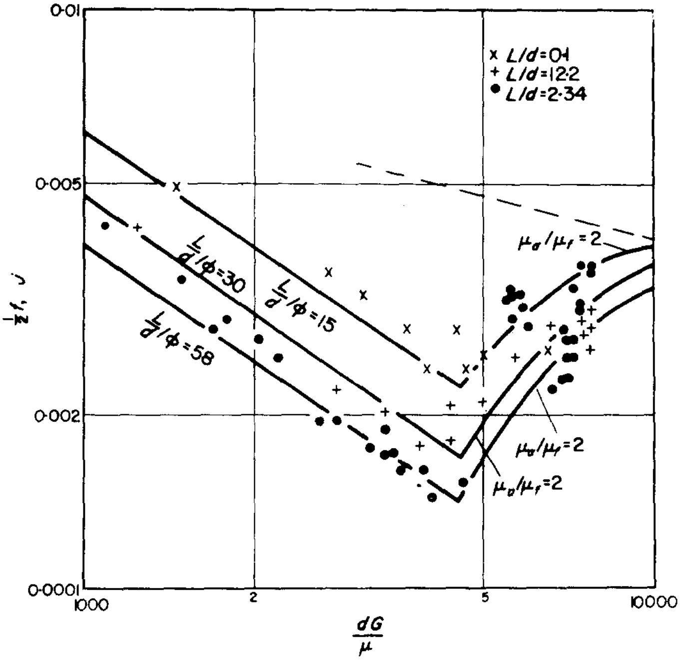


FIG. 10. Sherwood, Kiley and Mangsen—heat-transfer data.

Light heat transfer oil $(c\mu/k) = 68\text{–}90$, $\phi \doteq 4$.
$(\mu_a/\mu_f) = 1{\cdot}4\text{–}2{\cdot}2$.

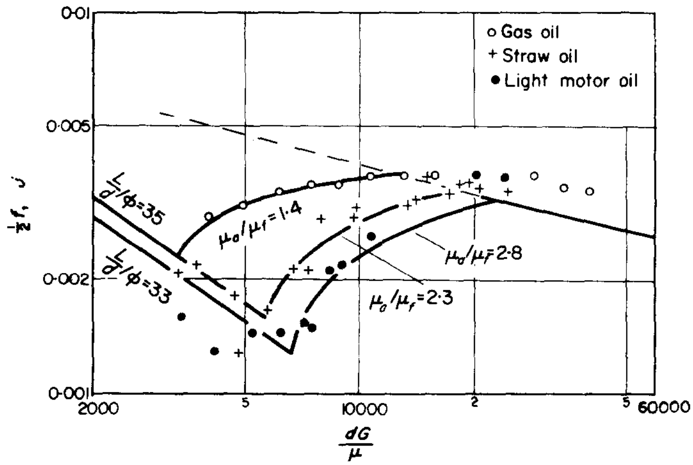
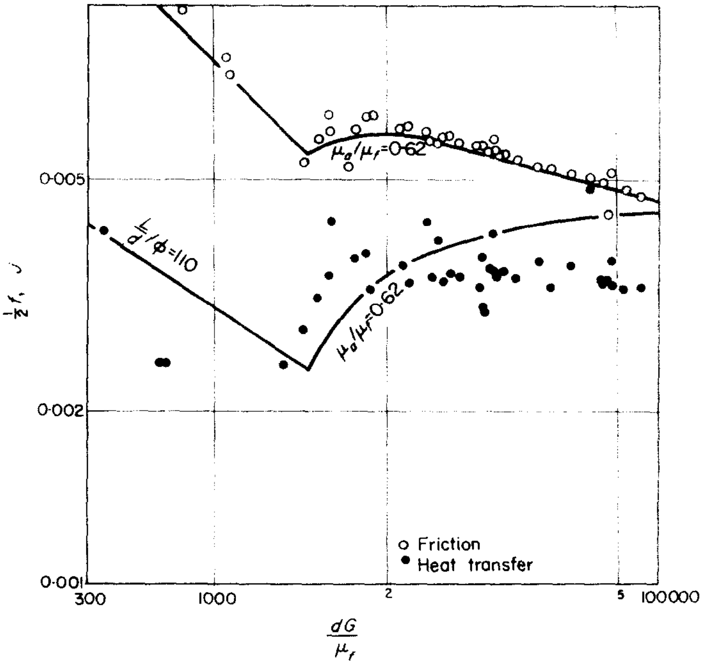


FIG. 11. Morris and Whitman—heat-transfer data, $L/d = 196$.

Gas oil	$(c\mu/k) = 25\text{–}\ 29.$	$(\mu_a/\mu_f) = 1{\cdot}3\text{–}1{\cdot}6$, $\phi \doteq 8$.
Straw oil	$(c\mu/k) = 40\text{–}\ 88.$	$(\mu_a/\mu_f) = 1{\cdot}4\text{–}2{\cdot}8$, $\phi \doteq 8$.
Light motor oil	$(c\mu/k) = 85\text{–}160.$	$(\mu_a/\mu_f) = 2{\cdot}4\text{–}3{\cdot}1$, $\phi \doteq 6$.



FIGS. 12, 13, 14, 15. Liquids in tubes—Cooling.

FIG. 12. McAdams and Keevil—heat-transfer and friction data.

Rabbeth I oil $(c\mu/k) = 85$–190, $\phi \doteq 1$.
$L/d = 110$ $(\mu_a/\mu_f) = 0{\cdot}59$–$0{\cdot}68$.

$$\frac{t_2 - t_1}{\Delta t_m} \frac{d}{4L} \left(\frac{c\mu}{k}\right)^{2/3},$$

the height of the ordinate is directly related to the temperature rise or fall of a fluid passing through an exchanger. The very marked reduction of the temperature rise of oils in the dip region is brought out in a striking manner. The effect of increasing the velocity on the temperature rise is shown to be very small for fluids in fully developed turbulent flow and thus indicates directly why the capacity of a heat exchanger can often be doubled with but little decrease in temperature rise. It should be remembered that the variable, Δt_m, is the mean temperature difference between the fluid and the surface, not the overall temperature difference between two fluids on opposite sides of the surface, so that the use of the temperature rise ordinate is limited to cases where only the one film resistance is considered.

An advantage of considerable convenience in using Fig. 16 in the turbulent region over previous plots is the reduction in the number of necessary physical properties from μ, k, and $(c\mu/k)^{1/3}$ to μ and $(c\mu/k)^{2/3}$. This is of particular help with gases, where k varies with temperature but $(c\mu/k)$ is independent of both temperature and pressure over moderate ranges. A list of values of $(c\mu/k)$ for gases is given by McAdams [37]. Convenient alignment charts for the viscosities of many liquids and gases at various temperatures are given by Genereaux (19] and for values of $(c\mu/k)$ of many liquids by Vernon [68].

2. HEAT TRANSFER AND FLUID FRICTION ACROSS TUBES

Single tubes. A comparison between heat transfer and friction for flow of fluids across a single cylinder is given by Fig. 17. While it has been shown by various investigators that the heat-transfer and friction coefficients vary markedly around the circumference, the present

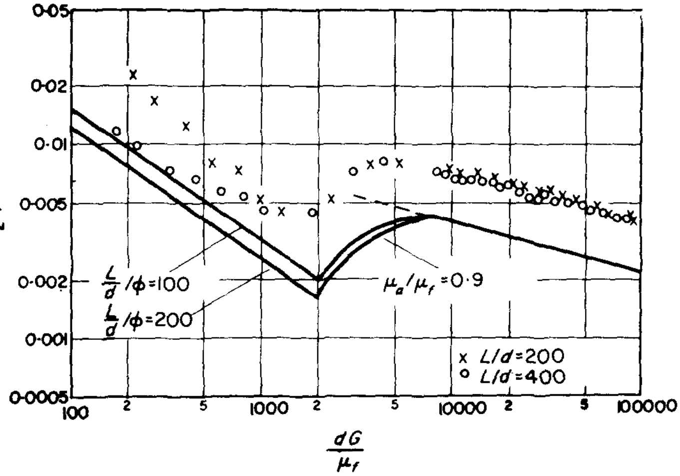


FIG. 13. Burbach—heat-transfer data.
Water $(c\mu/k) = 1·5–10$, $\phi \doteq 2$.
$d = 0·5$ cm.

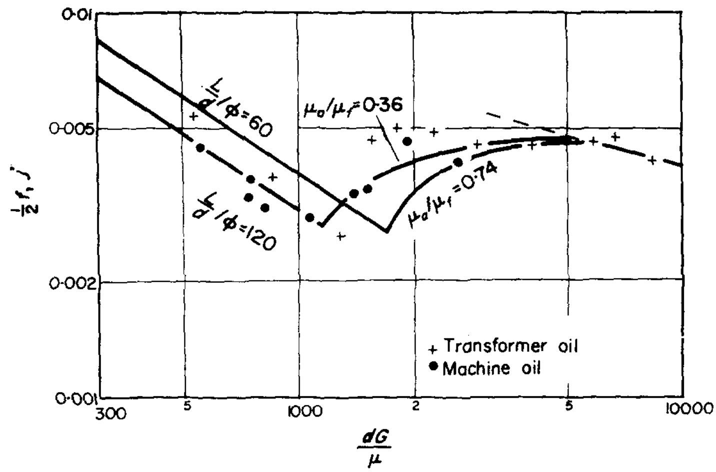


FIG. 14. Kraussold—heat-transfer data, $L/d = 117$.
Transformer oil $(c\mu/k) = 100–130$. $(\mu_a/\mu_f) = 0·66–0·83$, $\phi \doteq 2$.
Machine oil $(c\mu/k) = 300–500$. $(\mu_a/\mu_f) = 0·43–0·72$, $\phi \doteq 1$.

treatment is limited to the average overall effects. The curve for friction was obtained from a correlation by C. B. Shepherd [54] of experimental data from the literature, particularly those of Eisner [16] and later investigators. The curve for heat transfer was obtained from the correlation by W. H. McAdams [40] of experimental data for air on the basis of hd/k vs dG/μ. The ordinates were divided by the abscissas and by $(c\mu/k)^{1/3}$ to give the ordinates used herein: $(h/cG)(c\mu/k)^{2/3}$. The figure given by McAdams also contains a line for friction following a suggestion of Davis [10] which shows deviations between the friction and heat transfer similar to that of Fig. 17. It is concluded that the turbulence set up in the air stream by the cylinder causes a large share of the drag on it whereas only the surface friction is useful for transferring heat. At high Reynolds numbers the turbulence becomes disproportionately large.

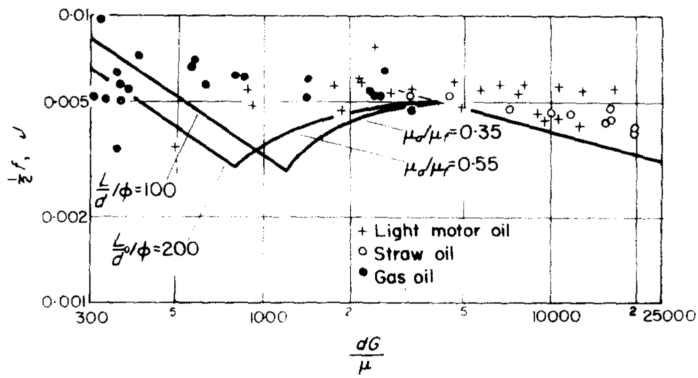


FIG. 15. Morris and Whitman—heat-transfer data, $L/d = 196$.

Gas oil	$(c\mu/k) = 22–41.$	$(\mu_a/\mu_f) = 0·72–0·84,\ \phi \doteq 4.$
Straw oil	$(c\mu/k) = 32–230.$	$(\mu_a/\mu_f) = 0·45–0·67,\ \phi \doteq 2.$
Light motor oil	$(c\mu/k) = 200–740.$	$(\mu_a/\mu_f) = 0·3–0·45,\ \phi \doteq 1.$

In an excellent treatment of this case, White [70] assumed that the surface friction at a plane surface under the same velocity conditions could be used to represent the skin friction or "tangential drag" on a cylinder, and that the remainder was "form drag" only. The skin friction was then shown to check the heat-transfer data on a plot of $(h/cG)(c\mu/k)$ vs (dG/μ), thus differing from Fig. 17 only by having $(c\mu/k)$ taken to the first power in place of the two-thirds.

Staggered tube banks. Data of the following investigators have been plotted on Fig. 18: Reiher [50], Rietschel [52], Carrier and Busey [4], Allen [1], Soule [58] and Dehn [11]. In the correlation of data on flow across staggered tube banks given by Fig. 18, it is apparent that the ratio of the clearance, d_s, between adjacent pipes in a row, to the pipe diameter, d_p, had practically no effect on the heat-transfer coefficients over a range of values of the ratio from 0·15 to 4, at least for Reynolds numbers greater than 2500, when the data were plotted as $(h/cG_m)(c\mu/k)^{2/3}$ vs d_pG_m/μ, where G_m is the maximum velocity (or velocity through the minimum area) and d_p is the diameter of the tubes. Plots were also made of the data as $(h/cG_m)(c\mu/k)^{2/3}$ vs d_sG_m/μ, and $(h/cG_a)(c\mu/k)^{2/3}$ vs dG_a/μ where d is the equivalent hydraulic diameter of the bank and G_a is the velocity based on the average cross-sectional area; but in these plots the data spread, and depended on the value of d_s/d_p. In obtaining the best line through the data it seemed reasonable to draw a curve through the points parallel to the single tube results, which extended over a much greater range of Reynolds numbers. This curve can be represented over the range of Reynolds numbers from 2000 to 40 000 by the following equation:

$$\frac{h}{cG_m}\left(\frac{c\mu}{k}\right)^{2/3} = j = 0·33\left(\frac{d_pG}{\mu}\right)^{-0·4}. \quad (18)$$

The friction data points were not included on the same figure, since the correlations found by Chilton and Genereaux [5] indicated that they would vary with the ratio d_s/d_p when plotted in a manner analogous to that used on the heat-transfer data, i.e. as $\frac{1}{2}f = R\rho/G_m^2 = (\Delta P\rho g/G_m^2)(d_s/\pi d_tN)$ vs d_pG_m/μ, where N = number of rows of tubes.

For banks of tubes on a *square arrangement*, only two sets of data were found; Reiher's results coincided with the line for single cylinders, while Dehn's few points were about 20 per cent lower. Dehn's data on staggered banks are also lower than Reiher's by about the same amount.

Résumé chart for fluids across tubes

A résumé chart for this case is given by Fig. 19. The heat-transfer line for staggered banks has been extrapolated as indicated by the dashed portions, although in the region of low Reynolds numbers where the flow is practically viscous, the ratio d_s/d_p may enter. Lines for pressure drop

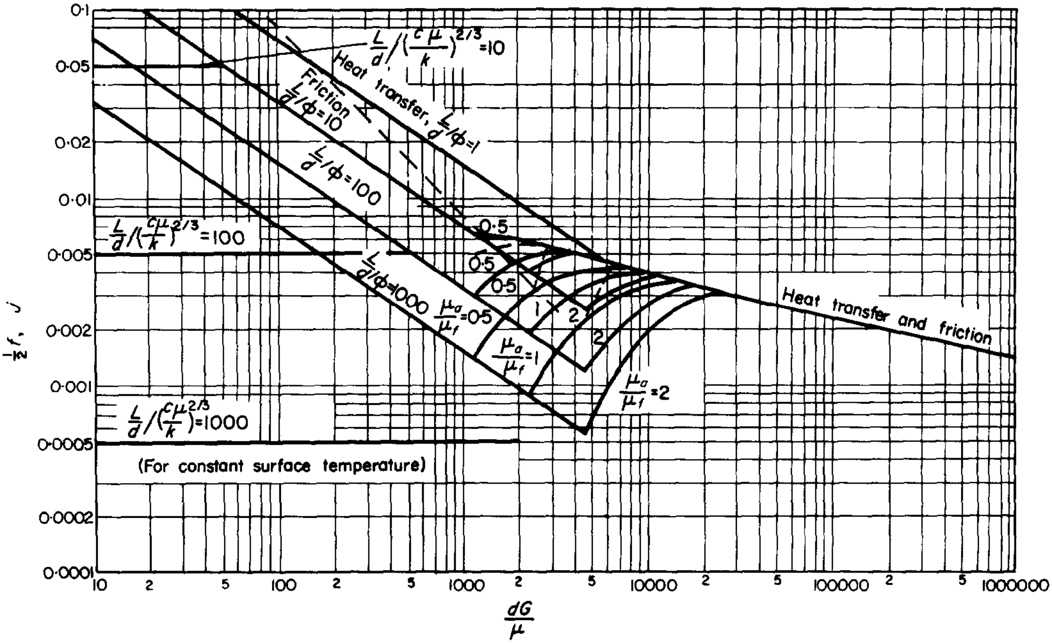


FIG. 16. Transfer processes in conduits.

Heat transfer: $j = \frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3} = \frac{t_2 - t_1}{\Delta t_m}\frac{d}{4L}\left(\frac{c\mu}{k}\right)^{2/3}.$

Mass transfer: $j = \frac{Kp_{gf}}{M}\left(\frac{\mu}{\rho k_d}\right)^{2/3} = \frac{p_2 - p_1}{\Delta p_m}\frac{p_{gf}}{p_g}\frac{d}{4L}\left(\frac{\mu}{\rho k_d}\right)^{2/3}.$

Friction: $\frac{1}{2}f = \frac{R\rho}{G^2} = \frac{\Delta P \rho g}{G^2}\frac{d}{4L}.$

$$\phi = (\mu_a/\mu_f)(1 + 0{\cdot}015\, Gr^{1/3})^3.$$

Gr, $(d^3\rho^2\beta\Delta t g/\mu^2)$;
h, heat-transfer coefficient;
K, molar mass-transfer coefficient;
G, mass velocity;
M, molar mass velocity;
d, equivalent diameter;
L, length;
R, frictional resistance;
ΔP, pressure drop;
p_{gf}, logarithmic mean partial pressure of inert gas in film;
p_g, inert gas pressure;
g, gravity;
c, specific heat;
μ_f, viscosity at film temperature;
μ_a, viscosity at average temperature;
k, thermal conductivity;
k_d diffusion coefficient;
ρ, density;
β, coefficient of expansion.

(Self-consistent units, e.g. lb, h, ft, P.c.u., degC.)

have been included for various ratios of d_s/d_p and again the solid portions indicate ranges covered by data, and the dashed portions extrapolations. These lines are based on the correlation of data by Chilton and Genereaux [5] expressed mathematically for the *turbulent region* as:

$$\frac{\Delta P \rho g}{G_m^2 N} = 1{\cdot}5\left(\frac{d_s G_m}{\mu}\right)^{-0{\cdot}2}. \qquad (19)$$

In terms of the friction factor used herein, equation (19) becomes:

$$\tfrac{1}{2}f = \frac{\Delta P \rho g}{G_m^2}\frac{d_s}{\pi d_p N} = \frac{1{\cdot}5}{\pi}\left(\frac{d_s}{d_p}\right)^{0{\cdot}8}\left(\frac{d_p G_m}{\mu}\right)^{-0{\cdot}2}. \qquad (20)$$

For the *viscous region* the results of Sieder and Scott [57] on two different spacing can be represented by the equation:

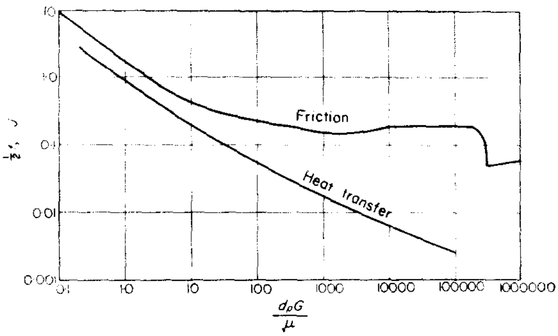


FIG. 17. Heat transfer and friction single cylinders.
Friction, $\frac{1}{2}f = R/\rho u^2$.
Heat transfer, $j = (h/cG)(c\mu/k)^{2/3}$.

$$\tfrac{1}{2}f = 2{\cdot}34 \left(\frac{d_p G_m}{\mu}\right)^{-1} \left(\frac{d_s}{d_p}\right)^{-1}. \qquad (21)$$

This equation was used to obtain the lines shown for friction in the viscous region. The opposite effect of the ratio d_s/d_p on friction in the viscous and turbulent regions suggests that although the ratio has no effect on heat transfer in the turbulent region, it probably enters in the viscous, as mentioned above.

It should be noted that the film temperature, t_f, should be used in computing Reynolds number for both heat transfer and friction in the turbulent region, but the film temperature, t_{vf}, for friction in the viscous region [5] as defined by equations (7) and (6), respectively.

The great contrast between the heat-transfer and friction lines in the turbulent region shows, particularly for large spaces between the tubes, that most of the resistance is due to eddy turbulence and is not useful for heat transfer. It is definitely shown that the use of a friction line for predicting heat transfer would be very unsafe for flow across tubes.

3. HEAT TRANSMISSION AND FLUID FRICTION AT PLANE SURFACES

Remarkable agreement between friction and heat-transfer data in both the viscous and turbulent regions is shown by Fig. 20, in which the lines represent friction results as correlated by Hopf [22] and the points, heat-transfer data of the following investigators: Jürges [25], Elias [17], and Fage and Falkner [18]. Both the heat-transfer and friction data in the *turbulent region* can be expressed by the equation:

$$\frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3} = \frac{R\rho}{G^2} = 0{\cdot}036 \left(\frac{LG}{\mu}\right)^{-0{\cdot}2}. \qquad (22)$$

This case had been previously used with the greatest success in developing theoretical analogies between heat transfer and friction. Prandtl [48] and Latzko [30] made frictional analogies

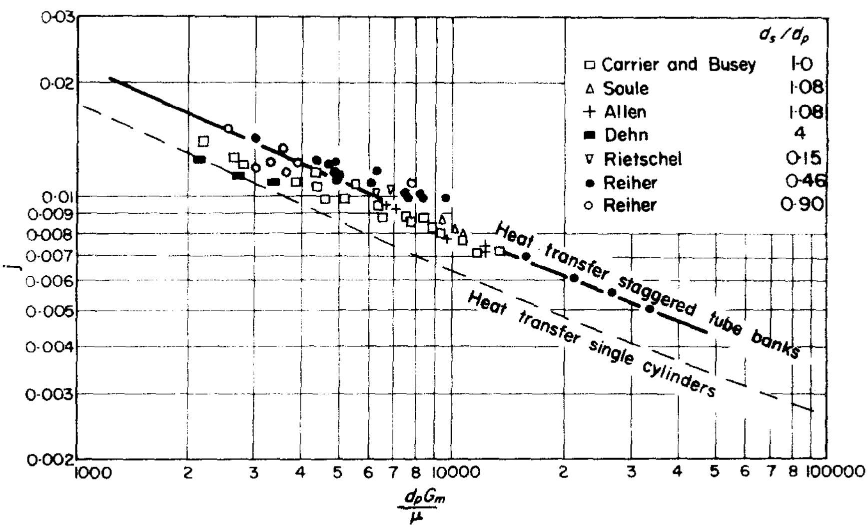


FIG. 18. Heat transfer, staggered tube banks.

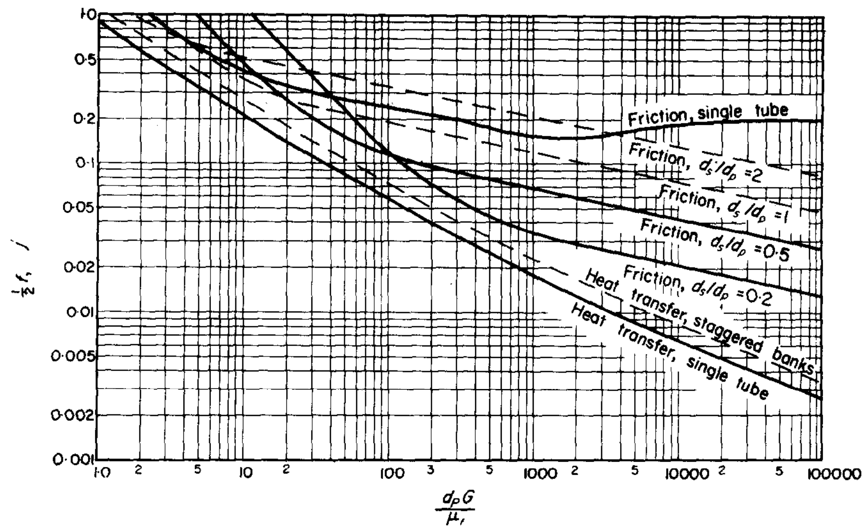


FIG. 19. Transfer processes, staggered tube banks.

Heat transfer: $$j = \frac{h}{cG_m}\left(\frac{c\mu}{k}\right)^{2/3} = \frac{t_2 - t_1}{\Delta t_m}\frac{d_s}{\pi d_p N}\left(\frac{c\mu}{k}\right)^{2/3}.$$

Mass transfer: $$j = \frac{Kp_{gf}}{M_m}\left(\frac{\mu}{\rho k_d}\right)^{2/3} = \frac{p_2 - p_1}{\Delta p_m}\frac{p_{gf}}{p_s}\frac{d_s}{\pi d_p N}\left(\frac{\mu}{\rho k_d}\right)^{2/3}.$$

Friction: $$\tfrac{1}{2}f = \frac{R\rho}{G_m^2} = \frac{\Delta P \rho g}{G_m^2}\frac{d_s}{\pi d_p N}.$$

h, heat-transfer coefficient;
K, molar mass-transfer coefficient;
G_m, mass velocity through minimum area;
M_m, molar mass velocity;
d_s, slit width between tubes;
d_p, outside tube diameter;
N, number of rows;
R, frictional resistance;
ΔP, pressure drop;
p_{gf}, logarithmic mean partial pressure of inert gas in film;
p_g, inert gas pressure;
g, gravity;
c, specific heat;
μ_f, film viscosity;
k, thermal conductivity;
k_d, diffusion coefficient;
ρ, density.

(Self-consistent units, e.g. lb, h, ft, P.c.u., degC.)

for the turbulent region similar to those for flow inside pipes. Pohlhausen [47] derived the following equation for the *viscous region*:

$$h = \frac{k}{L}\left(\frac{LG}{\mu}\right)^{1/2}\psi\left(\frac{c\mu}{k}\right), \qquad (23)$$

where L is the length of the plate in the direction of flow and the function $\psi(c\mu/k)$ can be approximated by $0{\cdot}66(c\mu/k)^{1/3}$.

Equation (23) can be written:

$$j = \frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3} = 0{\cdot}66\left(\frac{LG}{\mu}\right)^{-1/2}, \qquad (24)$$

which is also the equation of the friction line on Fig. 20,

$$\tfrac{1}{2}f = \frac{R\rho}{G^2} = 0{\cdot}66\left(\frac{LG}{\mu}\right)^{-1/2}. \qquad (25)$$

Thus Pohlhausen introduced mathematically for this case approximately the same function of the $(c\mu/k)$ group which has since been found necessary to correlate data for gases and liquids inside tubes. The exact agreement between heat-transfer and friction data in both the turbulent and viscous regions is particularly striking when compared to the results on flow inside and across tubes.

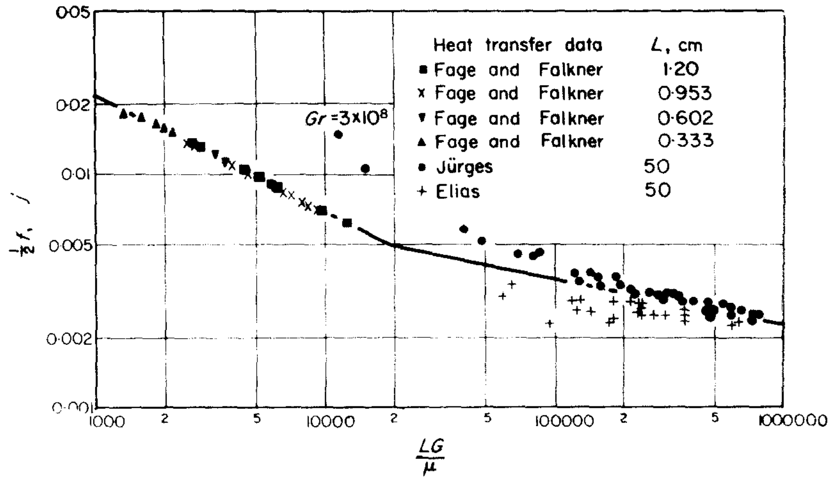


FIG. 20. Transfer processes, plane surfaces.

Friction: $\frac{1}{2}f = \frac{R\rho}{G^2}$.

Heat transfer: $j = \frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3}$

Mass transfer: $j = \frac{Kp_{gf}}{M}\left(\frac{\mu}{\rho k_d}\right)^{2/3}$.

h, heat-transfer coefficient;
K, molar mass-transfer coefficient;
G, mass velocity;
M, molar mass velocity;
L, length;
R, frictional resistance;
p_{gf}, logarithmic mean partial pressure of inert gas in film;
c, specific heat;
μ, film viscosity;
k, thermal conductivity;
k_d, diffusion coefficient;
ρ, density.

(Self-consistent units, e.g. lb, h, ft, P.c.u., degC.)

The high values of Jürges' results at low Reynolds numbers is due to the effect of free convection. The Grashof number, $(L^3\rho^2 g\beta\Delta t/\mu^2)$, is high, due to the large value of L, and amounts to around 300 000 000, whereas the Grashof number in Fage and Falkner's experiments was only at most 50 000 because of the small size of the heating surfaces used by them.

MASS TRANSMISSION AND FLUID FRICTION

It is hoped that a similar analogy with fluid friction can be extended to mass transmission, and that an equation similar in form to equation (2) involving mass-transfer rates, partial pressure differences, and the group $(\mu/\rho k_d)$, can be employed with the same ordinates as apply for heat transfer. A somewhat similar method has already been shown to give agreement with data inside tubes on the basis of an equation of the Prandtl type [6]. The equations given in the résumé charts are therefore suggested for preliminary estimation of mass transmission coefficients. Unfortunately, experimental values of μ and k_d are now available for only a few gas mixtures.

CONCLUSION

In the correlation of heat-transfer and friction results for this paper, an attempt has been made to study data observed under the widest possible range of conditions. For this reason the choice of the data included has been based mainly on their covering unique conditions, and a large amount of excellent data in the literature for flow in tubes was not correlated since they would have fallen in line with those selected. In constructing the résumé figures from the plotted data, several conditions were found where additional results should be obtained. These are as follows:

(*a*) For flow in tubes: heat-transfer and friction data in the dip region for both heating and cooling; heat transfer for heating viscous oils at a high enough range of Reynolds numbers to ensure the results being beyond the dip (in Figs. 8–11, only the two least viscous oils of Morris and Whitman covered a high enough range); heat transfer during heating and cooling in vertical and horizontal pipes in the streamline region under carefully varied conditions to determine the separate effects of free convection as a function of Grashof number, and of the radial change in the parabolic velocity distribution as a function of the viscosity change across the cross-section; heat transfer in the viscous region, particularly at very low Reynolds numbers, when the pipe wall temperature varies with length; velocity and temperature distributions during the heating and cooling of liquids covering a large range of values of $c\mu/k$ to assist in evaluating the functions of $c\mu/k$ and μ_a/μ_f to be used in heat-transfer equations. (*b*) Flow across tube banks: heat transfer and friction data at low Reynolds numbers for various tube spacings. (*c*) Plane surfaces: temperature and velocity distributions for the heating and cooling of liquids covering a wide range of $c\mu/k$. (*d*) Other types: heat-transfer and friction data on other types of heat-transfer surface, particularly various baffle arrangements in the shell side of heat exchangers.

In conclusion, the hope is again expressed that most of the investigations of heat transfer for forced convection in the future will include observations of friction data in the same apparatus and that these will be reported along with those for heat transfer.

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APPENDIX

The rate of heat transmission between a surface and a fluid passing over it in turbulent motion has long defied purely theoretical solution because of the lack of simple mathematical relationships for turbulent flow itself. In evaluating the friction between the moving fluid and the surface, empirical correlations of experimentally measured values by use of dimensionless groups of variables have been necessary in place of theoretically determined equations. These empirical relations for friction have been very successful in interpreting data for both gases and liquids on the same basis. Because the transfer of heat and the transfer of momentum are analogous processes, many relationships between them have been developed for the purpose of utilizing friction data in the prediction of heat-transfer rates.

Osborne Reynolds [51] pointed out in 1874 that the transmission of heat from a hot fluid to a surface was probably directly related to the fluid friction exerted by the fluid on the surface. In 1897, Reynolds, as quoted by Stanton [59], postulated that the "motion of heat" should be analogous to the "motion of momentum", and, using an early modification of the following equation for pressure drop inside a pipe:

$$\frac{\partial pd}{\partial L \rho u^2} = a\left(\frac{\mu}{d\rho u}\right)^n, \tag{26}$$

derived the equivalent of the following equation for heat exchange inside a pipe:

$$\frac{\partial td}{\partial L \Delta t} = a\left(\frac{\mu}{d\rho u}\right)^n, \tag{27}$$

where $\partial p/\partial L$ and $\partial t/\partial L$ are the rates of change of pressure and temperature, respectively, with length of the pipe of diameter, d, through which the fluid with density, ρ, and viscosity, μ, flows with average velocity, u; a and n are constants. Later Stanton [60] and Lanchester [29] stated the Reynolds theory more fully and expressed their results in the following equations:

$$h = \frac{cR}{u} \tag{28}$$

or

$$h = \tfrac{1}{2} fcG. \tag{29}$$

The friction factor from equation (1) was shown by Blasius [2] in 1913, by Stanton and Pannell [62] in 1914, and by many investigators since, to be a function of Reynolds number, dG/μ. Strangely enough, until recently no correlations of heat-transfer data have been made as h/cG vs Reynolds number, probably owing to the early introduction by Nusselt [44] of correlations as hd/k versus Reynolds number or Peclet number, (dcG/k).

For example, Prandtl [48] derived equation (28) independently in 1910 by showing that the fundamental equations for heat conduction and momentum transfer would be analogous where $c\mu/k = 1$. His expression for frictional resistance was, however,

$$R = \frac{a\mu u}{d}\left(\frac{d\rho u}{\mu}\right)^m, \tag{30}$$

so that his resulting expression for heat transfer was the equation obtained previously by Nusselt:

$$h = \frac{bk}{d}\left(\frac{d\rho u}{\mu}\right)^m, \tag{31}$$

where a, b, and m are constants.

Reynolds [59] predicted that the ratio of the thermal conductivity to the viscosity of the fluid would affect the heat-transfer coefficient. Prandtl, however, first mentioned the significance of the group, $c\mu/k$, in his comparison of the equations for heat conduction and momentum transfer. According to the kinetic theory of gases [33], the transfer of momentum is directly analogous to the transfer of energy, and $k = 1{\cdot}6\ c_v/\mu$. For diatomic gases where c_p/c_v is equal to 1·4, the theoretical value of $c\mu/k$ becomes 0·87. For air, the value of $c\mu/k$ based on experimental data is about 0·76, which is not far from unity, especially when compared with values for water between 2 and 10, and for oils up to 1000. It was shown by Stanton [61] from data of Pannell [45] that for air flowing in a heated pipe, the velocity and temperature distributions were almost coincident, and Elias [17] showed a similar relationship between the velocity and temperature fields for air flowing over a heated plate. For these cases, where $c\mu/k$ is almost equal to unity, equations (28) and (29) very nearly check the heat-transfer data for flow inside tubes and parallel to plane surfaces.

Prandtl [48] and Taylor [65] independently introduced a modification of equations (28) and (29) to apply to cases where $c\mu/k$ is not equal to unity. Their treatments considered a viscous film next to the solid surface in which pure conduction and laminar flow prevail, and a turbulent core in which the velocity and temperature fields coincide. Their resulting equation is:

$$h = \frac{\tfrac{1}{2} fcG}{1 - r + r(c\mu/k)}, \tag{32}$$

where r = ratio of the velocity at the film-core interface to the average velocity in the tube. Later Prandtl [49] showed from his theory of turbulence that the value of r should decrease with increasing Reynolds number, and this conclusion was shown to be roughly checked by Stender's [63] data for water in tubes as recalculated by Lawrence and Hogan [31], although, unfortunately, other factors such as water temperature and temperature difference also affected the value of r. Taylor [66] utilized a mathematical method based on the theory of velocity gradients in turbulent flow, to develop relationships for the temperature gradient. His analysis led to an infinite series which remains to be mathematically interpreted in order to be of assistance on this problem. Recently, Taylor [67] further discussed the original Reynolds analogy and showed that it is based on physical conditions not satisfied where heat flows between the pipe wall and the fluid, and thus the theory could not be expected to hold exactly under the usual conditions of heat transfer. Eagle and Ferguson [15] and later Murphree [43] considered a buffer zone between the viscous film and the turbulent core. Murphree assumed that in the buffer zone the degree of turbulence is a function of the distance from the surface, and obtained a table of values of hd/k as functions of dG/μ and $c\mu/k$. Data of Woolfenden [71] on temperature and velocity distributions for water flowing inside pipes, however, indicate a radical difference between these distributions in the turbulent core. For example, when the Reynolds number was about 80 000, the velocity in the turbulent core varied as the 0·15 power of distance from the wall, whereas the temperature varied only as the 0·06 power, indicating much less resistance to heat transfer than to momentum transfer in the core. As a result, the Prandtl equation would be expected to give low results when $c\mu/k$ differs considerably from unity. A derivation of the correct theoretical equation for these conditions would require a knowledge of the exact effect of $c\mu/k$ on the relation between the temperature and velocity distributions.

Other treatments of the relationship between heat transfer and fluid friction have been presented by Latzko [30], Lorenz [34], Schiller and Burbach [53], Tarassenko [64], and White [70]. Recently Margoulis [41] published a comprehensive review of the various theoretical papers and compared equation (29) with Nusselt's [44] data on gases in tubes. Margoulis plotted the data as h/cG versus Reynolds number and found good agreement between the lines for friction and the heat-transfer data, though, as seen from Fig. 3, his check with one point in the viscous region is a coincidence. For liquids, Margoulis employed the Prandtl equation and found a fair check with data on water, but did not try it on liquids with high values of $c\mu/k$, such as oils. Lorenz [35] in 1930 plotted both pressure drop and heat-transfer data from radiator tests and was possibly the first to use a plot of h/cG vs Reynolds number.

DISCUSSION

O. A. Hougen (written): In this paper for the first time, coefficients necessary for calculating heat transmission into any fluid flowing in a pipe have been presented on a single graph covering the turbulent, stream line and intermediate regions; in these correlations the dimensionless group h/cG is used instead of the usual Nusselt number hd/k; the viscosity values are taken at the average film temperatures instead of the temperature of the main stream, though the transition to turbulence occurs at a Reynolds number of 2300 based on the viscosity at the average main stream temperature—these appear to me to be the important contributions made.

It is my opinion that the analogy between friction and heat transmission has been overemphasized by many investigators. . . . Although there is nearly an exact analogy between the two processes for turbulent flow through pipes and flow parallel to plane surfaces, this similarity breaks down entirely for viscous flow, intermediate flow, and flow across tubes. There is, further, no analogy between the two processes for flow around bends and through contractions and restrictions. The analogy between heat flow and friction seems to be the exception rather than the rule.

. . . In view of the great difficulties involved in obtaining accurate data on heat transmission, particularly in the measurement of point and average temperatures and of the rate of fluid flow, it would be well for technical periodicals to restrict for publication only the experimental work of those who have had extensive experience in this type of work. The large amount of experimental work on heat transmission published by amateurs has made the task of correlations and evaluation of experimental data very difficult, as is well illustrated in the present paper.

E. N. Sieder (written): The general method proposed by Dr. Colburn is perhaps the most rational and covers the largest field of data yet published. The method brought out in his paper should allow more accurate predictions of heat transfer rates in the critical region. The use of the film temperature certainly seems logical for correlation of heat-transfer and pressure drop data in the region of viscous flow. However, I am not entirely certain that film temperatures should be used in correlating heat-transfer and pressure drop data in the region of turbulent flow. . . . A large volume of data on cooling of fluids inside of tubes, which I have correlated on the basis of main stream properties, shows an excellent check with the published data on heating also correlated in the same manner. I would advocate the use of two distinct curves—one for cooling and one for heating—with the fluid properties taken at the main stream temperatures. . . .

I have found that recent transfer tests taken with values of *Re* below 2300 show that the use of d_p gives better correlation than d. However, there is some indication of a change in heat-transfer rates with different values of d_s and d_p.

LINCOLN T. WORK (written): The two papers from the division of fundamental chemical engineering research at the Experimental Station of the du Pont Company represent significant contributions to the field of fluid flow and heat transfer. They are of a type none too common and greatly needed in chemical engineering, namely in the correlation of diversified experimental results. . . .

There are other fields of chemical engineering in which this type of correlation is not only essential as a mode of attack but is an especially difficult one. The field of heterogeneous systems often involving the dynamics of reactions offers more complicated variables even than are offered in fluid flow and heat transfer. In many cases the complexity is so great that relations of a statistical type are used. Here both diversity and number of tests are essential to the drawing of accurate conclusions.

A. P. COLBURN (written): The kind criticisms of Professor Hougen, and of others who have discussed the paper, both at the meeting and through personal communications, have prompted the author to make several revisions in the paper as originally presented, and these are included in the form here published.

Résumé—Une méthode générale pour la corrélation des données de transport de chaleur par convection forcée est proposée, qui consiste à porter, en fonction du nombre de Reynolds, un groupe sans dimensions représentant les données mesurées expérimentalement à partir desquelles les coefficients de transport de chaleur de film seraient calculés, c'est-à-dire, $[(t_1 - t_2)/\Delta t_m](S/A)$, ou son équivalent, h/cG, multiplié par le groupe $(c\mu/K)$ élevé à la puissance deux tiers. Des données sont citées à partir de la littérature qui montrent que les diagrammes résultants des données de transport de chaleur pour l'écoulement parallèle à des surfaces planes et pour l'écoulement entièrement turbulent dans des tubes, coïncident (lorsque les propriétés sont prises à la température de "film") avec les meilleurs renseignements sur la perte de charge portée de la façon ordinaire, sous forme du coefficient de perte de charge linéique

$$\tfrac{1}{2} f = \frac{\Delta P g}{\rho u^2} \cdot \frac{S}{A} = \frac{R}{\rho u^2}$$

en fonction du nombre de Reynolds. Cependant, pour l'écoulement perpendiculaire aux tubes, les coefficients de perte de charge et de transport de chaleur diffèrent, les coefficients de perte de charge étant plus élevés.

Les équations utilisées avec succès pour représenter les données de transport de chaleur dans un écoulement laminaire dans les tubes ont été modifiées pour des tracés avec les mêmes coordonnées que celles pour l'écoulement turbulent; et une modification quantitative est suggérée pour l'effet de la convection libre à faible vitesse en ajoutant une fonction du groupe $(d^3\rho^2\beta\Delta t g/\mu^2)$. On voit qu'il n'y a pas de relation entre le transport de chaleur et le frottement dans la région visqueuse.

On montre que la méthode de corrélation proposée ici est particulièrement valable dans la région de transition entre l'écoulement laminaire et l'écoulement turbulent dans les tubes, puisque les facteurs de transport de chaleur peuvent présenter des "creux" analogues à ceux pour le frottement. Les variables de contrôle dans cette région sont discutées complètement à la lumière données connues.

Аннотация—Предлагается общий метод обработки экспериментальных данных по теплообмену при вынужденной конвекции, заключающийся в представлении этих данных в виде безразмерных комплексов в зависимости от числа Рейнольдса, откуда вычисляется крэффициент теплообмена $[(t_1 - t_2)/\Delta t_m](S/A)$ или его эквивалент h/cG, умноженный на комплекс $(c\mu/k)$ в степени 2/3. Обработка литературных данных, показывает, что результаты опытов по теплообмену в плоско-параллельном потоке и при полностью развитом турбулентном течении внутри труб совпадают (при отнесении физических параметров к определяющей температуре) с самыми надежными данными по гидравлическому сопротивлению, построенными обычным методом в виде зависимости коэффициента трения

$$\tfrac{1}{2} f = \frac{\Delta P g}{\rho u^2} \frac{S}{A} = \frac{R}{\rho u^2}$$

от чисел Рейнольдса. Однако, при обтекании труб под прямым углом коэффициенты трения и теплообмена неодинаковы (коэффициент трения выше).

Уравнения, с успехом применяемые для обработки данных по теплообмену при ламинарном течении внутри трубы, модифицированы таким образом, что можно использовать те же координаты, что и при обработке данных для турбулентного течения. Предложен способ количественного учета свободной конвекции при низких скоростях путем введения комплекса $(d^3\rho^2\beta\Delta t g/\mu^2)$. Очевидно, что для вязких течений нет зависимости между коэффициентом теплообмена и трения.

Показано, что предложенный метод обобщения данных может быть успешно использован для области перехода от ламинарного течения к турбулентному в трубах, поскольку ход кривых для коэффициентов теплообмена и трения может быть аналогичен. Определяющие переменные для этой области подробно рассмотрены в свете имеющихся данных.